

Broué's Abelian Defect Group Conjecture for the Tits Group

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Abstract

In this paper we prove that Broué's abelian defect group conjecture holds for the Tits group ${}^2F_4(2)'$. Also we prove that under certain conditions we are able to lift derived equivalences and use this to prove Broué's conjecture for the group ${}^2F_4(2)$.

1 Introduction

Much of modern representation theory is concerned with the relationship between representations of a group and those of its subgroups. Broué's abelian defect group conjecture attempts to understand this relationship using derived equivalences and can be stated as follows:

Conjecture 1.1 (Broué's Abelian Defect Group Conjecture). *Let k be an algebraically closed field of prime characteristic p and G be a finite group. If A is a block, of the group algebra kG , with an abelian[†] defect group D , and B is the Brauer correspondent of A , a block of the group algebra $kN_G(D)$, then $D^b(\text{mod-}A)$ and $D^b(\text{mod-}B)$ are equivalent as triangulated categories.*

This conjecture is already known in many cases, for example when the defect group is cyclic, see [10], or A is a principal block with defect group $C_3 \times C_3$, see [7].

In many cases, where Broué's conjecture is still unknown, there is known to be a stable equivalence of Morita type between A and B , which can be seen as a consequence of the derived equivalence as the stable module category is a canonical quotient of the derived category, see [4]. This and an idea of Okuyama's [11] led Rickard to prove the following in his paper [6]:

Theorem 1.2 (Rickard's theorem). *Let A and B be finite-dimensional symmetric algebras,*

$$F : \text{mod-}A \rightarrow \text{mod-}B$$

[†]The abelian condition is not well understood but it is essential as the principal block of $kSz(8)$ in characteristic 2 has a non-abelian defect group and is not derived equivalent to its Brauer correspondent. However, there are still some interesting cases where "Broué's Conjecture" holds and the defect group is not abelian.

be an exact functor inducing a stable equivalence of Morita type and let S_1, \dots, S_n be a set of representatives for the isomorphism classes of simple A -modules.

If there are objects X_1, \dots, X_n of $D^b(\text{mod-}B)$ such that X_i is stably isomorphic to $F(S_i)$, for each $1 \leq i \leq n$, and such that the following are satisfied:

$$(a) \text{ Hom}_{D^b(B)}(X_i, X_j[m]) = 0 \text{ for } m < 0$$

$$(b) \text{ Hom}_{D^b(B)}(X_i, X_j) = \begin{cases} 0 & \text{if } i \neq j \\ k & \text{if } i = j \end{cases}$$

$$(c) X_1, \dots, X_n \text{ generate } D^b(\text{mod-}B) \text{ as a triangulated category}$$

then $D(\text{Mod-}A)$ and $D(\text{Mod-}B)$ are equivalent as triangulated categories. \square

In §3 we use this theorem to prove that Broué's conjecture holds for the Tits group.

Following Al-Nofayee [1] we make the following definition concerning the objects in Rickard's theorem:

Definition 1.3. *Given an algebra A , we say the objects X_1, \dots, X_n of $D^b(\text{mod-}A)$ are a cohomologically schurian set of generators if they satisfy the following conditions:*

$$(a) \text{ Hom}_{D^b(A)}(X_i, X_j[m]) = 0 \text{ for } m < 0$$

$$(b) \text{ Hom}_{D^b(A)}(X_i, X_j) = \begin{cases} 0 & \text{if } i \neq j \\ k & \text{if } i = j \end{cases}$$

$$(c) X_1, \dots, X_n \text{ generate } D^b(\text{mod-}A) \text{ as a triangulated category}$$

If we take k to be a group of characteristic p , \tilde{G} to be a group with a normal subgroup G , \tilde{N} to be a group with a normal subgroup N , $H = \tilde{G}/G \cong \tilde{N}/N$, A to be a block of kG and B to be a block of kN both stable under the action of H , set $\tilde{A} = A \uparrow^{\tilde{G}}$ and $\tilde{B} = B \uparrow^{\tilde{N}}$, note that \tilde{A} and \tilde{B} are blocks of $k\tilde{G}$ and $k\tilde{N}$ respectively, and list the simple A -modules as $s_1, \dots, s_m, s_{11}, \dots, s_{1k_1}, s_{21}, \dots, s_{2k_2}, \dots, s_{r1}, \dots, s_{rk_r}$ such that H fixes s_1, \dots, s_m and for each $1 \leq i \leq r$ permutes s_{i1}, \dots, s_{ik_i} , then in §4 we prove that:

Theorem 1.4. *Let $\tilde{F} : \overline{\text{mod}} - \tilde{A} \rightarrow \overline{\text{mod}} - \tilde{B}$ and $F : \overline{\text{mod}} - A \rightarrow \overline{\text{mod}} - B$ are stable equivalences of Morita type such that*

$$F(-) \uparrow^{\tilde{N}} \cong \tilde{F}(- \uparrow^{\tilde{G}}), \quad (1)$$

and $X_1, \dots, X_m, X_{11}, \dots, X_{1k_1}, \dots, X_{r1}, \dots, X_{rk_r} \in D^b(\text{mod} - B)$ is a cohomologically schurian set of generators such that H permutes X_{i1}, \dots, X_{ik_i} for $1 \leq i \leq r$ and fixes X_1, \dots, X_m , $X_{ij} \cong F(s_{ij})$ for $1 \leq i \leq r$ and $1 \leq j \leq k_r$, and $X_i \cong F(s_i)$ for $1 \leq i \leq m$. Then \tilde{A} and \tilde{B} are derived equivalent. \square

Finally, in §5, we combine the results of §3 and §4 to prove that Broué's conjecture holds for the group ${}^2F_4(2)$ and in the appendix we give the Loewy layers of the modules required in §3 and §5.

When we refer to programmes we are referring to the computer package MAGMA [8].

2 Notation

Throughout this paper we use the following notation. Let A be a ring, we denote by 1_A , $Z(A)$ and A^\times the unit element, the centre and the set of units of A respectively. The category of left A -modules is denoted by $\text{mod-}A$ and the category of right modules is denoted $A\text{-mod}$, unless otherwise stated all A -modules are assumed to be left A -modules. Also, we denote the category of projective A -modules by $\text{Proj-}A$ and the category of finitely-generated projective A -modules by P_A .

We always take G to be a group, p to be a prime number and (\mathcal{O}, K, k) to be a p -modular splitting system for all subgroups of G , that is, \mathcal{O} is a complete discrete valuation ring of rank one with quotient field K of characteristic 0 and residue field $k = \mathcal{O}/\text{rad}(\mathcal{O})$ of characteristic p such that K and k are splitting fields for all subgroups of G . When we talk of an $\mathcal{O}G$ -module we mean a finitely generated right $\mathcal{O}G$ -module which is free as an \mathcal{O} -module. Given an $\mathcal{O}G$ -module M we have a kG -module $M/[M.\text{rad}(\mathcal{O})]$, we often abuse notation and also denote this by M .

Our complexes will be *cochain* complexes, so the differentials will have degree 1 and if

$$X = \dots \rightarrow X^{i-1} \xrightarrow{d^{i-1}} X^i \xrightarrow{d^i} X^{i+1} \rightarrow \dots$$

is a cochain complex, then $X[m]$ denotes X shifted m places to the left, that is $X[m]^i = X^{i+m}$ and $d^i[m] = (-1)^m d^{i+m}$.

If A is an additive category then $K(A)$ will be the chain homotopy category of cochain complexes over A , $K^-(A)$ will be the full subcategory consisting of complexes X that are bounded above, i.e. $X^i = 0$ for $i \gg 0$, and $K^b(A)$ will be the full subcategory of bounded complexes, i.e. complexes X with $X^i = 0$ for all but finitely many i .

If A is an abelian category then $D(A)$ will be the derived category of cochain complexes over A , and $D^-(A)$ and $D^b(A)$ will be the full subcategories of complexes that are bounded above and bounded respectively.

For a natural number n , C_n denotes the cyclic group of order n and for a subgroup E of $\text{Aut}(G)$, $G \rtimes E$ denotes the semi-direct product.

3 Broué's Conjecture for the Tits Group

In this section we take G to be the simple group ${}^2F_4(2)'$, which is the derived subgroup of ${}^2F_4(2)$ and is often referred to as the Tits group, its order is $17971200 = 2^{11}3^35^213$. We also take k to be an algebraically closed field of characteristic p . We prove that Broué's conjecture holds for G .

Theorem 3.1. *It is enough to prove the conjecture for the principal block in characteristic $p = 5$.*

Proof. Using GAP[3] we see that non-principal blocks have trivial defect groups, so we only have to consider principal blocks. Sylow 2-subgroups and Sylow 3-subgroups are not abelian so there is nothing to prove. A Sylow 13-subgroup is

cyclic and Broué's conjecture has been proven for cyclic defect groups in [10]. The only remaining case to check is characteristic 5 where we do have an abelian Sylow 5-subgroup $C_5 \times C_5$. \square

We now take k to be an algebraically closed field of characteristic 5. The principal block, $A = ekG$, of kG has elementary abelian defect group P of order 25, we set $N = N_G(P)$, $B = fkG$ to be the principal block of kN and we prove that Broué's conjecture holds for A . Note that we are taking e and f to be the block idempotents of A and B respectively.

There are 14 simple kG -modules in A which we shall denote by S_1, \dots, S_{14} . To prove our result we shall find 14 complexes X_1, \dots, X_{14} which satisfy the conditions of Rickard's theorem 1.2. First we have to find a stable equivalence, in this case the equivalence is simply restriction since $C_G(Q) = C_N(Q)$ for every non-trivial subgroup $Q \leq P$. Therefore, if we take C to be $ekGf$ considered as a complex concentrated in degree zero, then C induces a stable equivalence between A and B . Moreover if we take RG_i to be the Green correspondant of S_i then:

$$S_i \otimes C : \dots \rightarrow 0 \rightarrow RG_i \oplus (proj_s) \rightarrow \dots$$

where the non-zero term is in degree zero.

We now want to construct complexes X_1, \dots, X_{14} such that X_i is stably isomorphic to $S_i \otimes_A C$ and which satisfy the conditions (a)-(c) of Rickard's theorem. To define the X_i 's we shall describe the complex P_i in $K^b(P_B)$ such that we have a triangle

$$P_i \rightarrow S \otimes_A C \rightarrow X_i \rightarrow P[1]$$

First we found an isotypy between A and B and hence found that from this isotypy we have the perfect isometry:

$$\begin{pmatrix} \chi_1 \\ \chi_{26a} \\ \chi_{26b} \\ \chi_{27a} \\ \chi_{27b} \\ \chi_{78} \\ \chi_{351a} \\ \chi_{351b} \\ \chi_{351c} \\ \chi_{624a} \\ \chi_{624b} \\ \chi_{702a} \\ \chi_{702b} \\ \chi_{1728} \\ \chi_{2048a} \\ \chi_{2048b} \end{pmatrix} \longrightarrow \begin{pmatrix} \phi_{1a} \\ \phi_{1c} \\ \phi_{1e} \\ \phi_{2f} \\ \phi_{2e} \\ \phi_{3a} \\ -\phi_{24b} \\ \phi_{1b} \\ -\phi_{24a} \\ -\phi_{1d} \\ -\phi_{1f} \\ \phi_{2a} \\ \phi_{2d} \\ \phi_{3b} \\ -\phi_{2b} \\ -\phi_{2c} \end{pmatrix}$$

where the subscripts denote the degree of the characters. This perfect isometry is suspected to be the map induced by the character χ of a complex $K \otimes_{\mathcal{O}} X$,

where X is a tilting complex of $e\mathcal{O}G\text{-}f\mathcal{O}N$ -bimodules. We also found the character χ_C of the complex C .

Using decomposition matrices we can calculate the map on Grothendieck groups $K_0(\text{mod-}A) \rightarrow K_0(\text{mod-}B)$ induced by the character χ of $k \otimes_{\mathcal{O}} X$. Assuming the tilting complex X lifts our complex C we have a triangle

$$Q \rightarrow C \rightarrow k \otimes_{\mathcal{O}} X \rightarrow Q[1]$$

where Q is a bounded complex of projective $ekG\text{-}kN$ -bimodules. We then have a map $K_0(\text{mod-}A) \rightarrow K_0(P_B)$ induced by $\chi - \chi_C$ and the image of the simple A -module S_i will then be the image in $K_0(P_B)$ of the complex P_i required to define X_i and hence this can act as a guide in choosing the terms of P_i .

We used the method outlined above to find the following complexes:

$$X_1 : \cdots \rightarrow RG_1$$

$$X_2 : \cdots \rightarrow RG_2$$

$$X_3 : \cdots \rightarrow RG_3$$

$$X_4 : \cdots \rightarrow P_{11} \rightarrow P_{14} \rightarrow RG_4$$

$$X_5 : \cdots \rightarrow P_7 \rightarrow P_{13} \rightarrow P_{13} \oplus P_7 \rightarrow P_7 \oplus P_{12} \oplus P_{10} \rightarrow P_{10} \oplus P_{12} \oplus P_2 \rightarrow \\ \rightarrow P_2 \oplus P_{14} \rightarrow RG_5$$

$$X_6 : \cdots \rightarrow P_2 \rightarrow P_2 \oplus P_{14} \rightarrow P_{11} \oplus P_7 \rightarrow RG_6$$

$$X_7 : \cdots \rightarrow P_7 \rightarrow P_{12} \rightarrow P_{10} \rightarrow P_7 \rightarrow P_3 \rightarrow RG_7$$

$$X_8 : \cdots \rightarrow P_7 \rightarrow P_{10} \rightarrow P_{12} \rightarrow P_7 \rightarrow P_5 \rightarrow RG_8$$

$$X_9 : \cdots \rightarrow P_2 \rightarrow P_{14} \rightarrow P_{14} \oplus P_7 \rightarrow P_9 \oplus P_8 \oplus P_7 \rightarrow RG_9$$

$$X_{10} : \cdots \rightarrow P_7 \rightarrow P_7 \oplus P_5 \oplus P_3 \rightarrow P_3 \oplus P_5 \oplus P_{13} \rightarrow P_{13} \oplus P_{13} \rightarrow \\ \rightarrow P_{13} \oplus P_{12} \oplus P_{10} \rightarrow RG_{10}$$

$$X_{11} : \cdots \rightarrow P_{13} \rightarrow P_7 \oplus P_5 \oplus P_{10} \rightarrow P_{10} \oplus P_{12} \rightarrow P_2 \oplus P_{13} \rightarrow P_5 \oplus P_{14} \rightarrow \\ \rightarrow P_{11} \oplus P_{10} \rightarrow RG_{11}$$

$$X_{12} : \cdots \rightarrow P_{13} \rightarrow P_7 \oplus P_3 \oplus P_{12} \rightarrow P_{10} \oplus P_{12} \rightarrow P_2 \oplus P_{13} \rightarrow P_3 \oplus P_{14} \rightarrow \\ \rightarrow P_{11} \oplus P_{12} \rightarrow RG_{12}$$

$$X_{13} : \cdots \rightarrow P_7 \rightarrow P_{10} \rightarrow P_{12} \rightarrow P_7 \rightarrow P_9 \oplus P_5 \rightarrow P_{13} \rightarrow RG_{13}$$

$$X_{14} : \cdots \rightarrow P_7 \rightarrow P_{12} \rightarrow P_{10} \rightarrow P_7 \rightarrow P_8 \oplus P_3 \rightarrow P_{13} \rightarrow RG_{14}$$

where in each case RG_i is in degree zero. Note that enough information is given in the appendix to determine these complexes uniquely up to isomorphism.

Finally, we want to prove that these X_i 's satisfy the conditions of Rickard's theorem and hence it will follow that Broué's conjecture holds for A .

First we know that the X_i 's are stably isomorphic to $S_i \otimes_A C$ by construction and then we prove that conditions (a) and (b) hold using the program contained in [9]. All that is left to prove is that the X_i 's generate $D^b(B)$ as a triangulated category. Note, while reading the following proof one should refer to the appendix where I have given the Loewy structure of the homology of the complexes X_i .

Lemma 3.2. *Let \mathcal{T} be the triangulated category generated by the X_i 's, then $\mathcal{T} = D^b(B)$.*

Proof. The proof of this theorem consists of taking each complex X_i and using the short exact sequences given in appendix B to prove that every composition factor of their homology, except one, is in \mathcal{T} . It then follows that every composition factor is in \mathcal{T} and we eventually conclude that \mathcal{T} contains every simple B -module. The details are as follows.

First, as X_1 , X_2 and X_3 have homologies concentrated in degree zero, we have $H^0(X_1) = 1 \in \mathcal{T}$, $H^0(X_2) = 4 \in \mathcal{T}$ and $H^0(X_3) = 6 \in \mathcal{T}$.

$H^{-1}(X_4) \cong 1 \in \mathcal{T}$ so $H^{-2}(X_4) \in \mathcal{T}$ and it follows from short exact sequence (B.1), in appendix B, that $11 \in \mathcal{T}$.

$H^{-2}(X_9) \cong 1 \in \mathcal{T}$ and $H^{-3}(X_9) \cong H^{-2}(X_4) \in \mathcal{T}$ so $H^{-4}(X_9) \in \mathcal{T}$ and $2 \in \mathcal{T}$ by short exact sequence (B.2).

$H^{-3}(X_6) \cong H^{-4}(X_9) \in \mathcal{T}$ so $H^{-2}(X_6) \in \mathcal{T}$ and $14 \in \mathcal{T}$ by short exact sequence (B.3).

$H^{-1}(X_{13}) \cong 4 \in \mathcal{T}$, $H^{-4}(X_{13}) \cong H^{-2}(X_4) \in \mathcal{T}$ and $H^{-5}(X_{13}) \cong H^{-4}(X_9) \in \mathcal{T}$, so $H^{-6}(X_{13}) \in \mathcal{T}$.

$H^{-3}(X_8) \cong H^{-2}(X_4)$, $H^{-4}(X_8) \cong H^{-3}(X_6)$ and $H^{-5}(X_8) \cong H^{-6}(X_{13})$, therefore, $H^{-2}(X_8) \in \mathcal{T}$ and thus by short exact sequence (B.5) we have $9 \in \mathcal{T}$.

$H^{-1}(X_{14}) \cong 6 \in \mathcal{T}$, $H^{-4}(X_{14}) \cong H^{-2}(X_4) \in \mathcal{T}$ and $H^{-5}(X_{14}) \cong H^{-4}(X_9) \in \mathcal{T}$, so $H^{-6}(X_{14}) \in \mathcal{T}$.

$H^{-3}(X_7) \cong H^{-2}(X_4)$, $H^{-4}(X_7) \cong H^{-3}(X_6)$ and $H^{-5}(X_7) \cong H^{-6}(X_{14})$, therefore, $H^{-2}(X_7) \in \mathcal{T}$ and thus by short exact sequence (B.4) we have $8 \in \mathcal{T}$.

All the composition factors of $H^{-3}(X_5)$, $H^{-4}(X_5)$ and $H^{-5}(X_5)$ have been shown to be in \mathcal{T} and so we must have $H^{-6}(X_5) \in \mathcal{T}$, hence by short exact sequence (B.6), $7 \in \mathcal{T}$.

The composition factors of $H^{-4}(X_{12})$ and $H^{-5}(X_{12})$ have been shown to be in \mathcal{T} and so $H^{-6}(X_{12}) \in \mathcal{T}$, it now follows from short exact sequences (B.7)-(B.9) that $N_3 \in \mathcal{T}$, 3 occurs once as a composition factor of N_3 and the other composition factor 9 is in \mathcal{T} , hence $3 \in \mathcal{T}$.

Similarly, the composition factors of $H^{-4}(X_{11})$ and $H^{-5}(X_{11})$ have been shown to be in \mathcal{T} and so $H^{-6}(X_{11}) \in \mathcal{T}$, it now follows from short exact sequences (B.10)-(B.12) that $N_6 \in \mathcal{T}$, 5 occurs once as a composition factor of N_6 and the other composition factor 8 is in \mathcal{T} , hence $5 \in \mathcal{T}$.

The composition factors of $H^{-1}(X_{10})$, $H^{-2}(X_{10})$, $H^{-3}(X_{10})$ and $H^{-4}(X_{10})$ have been shown to be in \mathcal{T} and so $H^{-5}(X_{10}) \in \mathcal{T}$. The short exact sequences (B.13) and (B.14) now imply that $N_8 \in \mathcal{T}$ and the only composition factor of N_8 not already shown to be in \mathcal{T} is 12 which occurs just once, therefore $12 \in \mathcal{T}$.

$H^{-5}(X_8) \in \mathcal{T}$ has 13 as a composition factor with multiplicity 1 and all other composition factors are in \mathcal{T} , so $13 \in \mathcal{T}$.

Finally, 10 is a composition factor of $H^{-5}(X_7)$ with multiplicity one and is the only simple not already shown to be in \mathcal{T} , hence we must have $10 \in \mathcal{T}$.

As we have shown all simple B -modules are contained in \mathcal{T} it follows that $\mathcal{T} = D^b(B)$. \square

Theorem 3.3. *Broué's conjecture holds for A .*

Proof. We have shown above that the X_i 's satisfy the conditions of Rickard's theorem and hence the result follows from Rickard's theorem itself. \square

There is actually a refinement of Broué's conjecture, due to Rickard. I refer the reader to Rickard's paper [5] for the definition of a splendid equivalence and state the refinement as:

Conjecture 3.4 (Broué/Rickard Conjecture). *If G is a finite group and A is a block of kG with an abelian defect group D and Brauer correspondent B , then there is a splendid equivalence between A and B .*

In [9] Holloway proves a variant of Rickard's theorem:

Theorem 3.5. *Suppose C is a complex of A - B -bimodules that induces a splendid stable equivalence between A and B and let $\{S_1, \dots, S_n\}$ be a set of representatives for the isomorphism classes of simple A -modules. If there are objects X_1, \dots, X_n of $D^b(B)$ such that, for each $1 \leq i \leq n$, X_i is stably isomorphic to $S_i \otimes_A C$, and such that:*

$$(a) \operatorname{Hom}_{D^b(B)}((X_i, X_j[m]) = 0 \text{ for } m < 0$$

$$(b) \operatorname{Hom}_{D^b(B)}((X_i, X_j) = \begin{cases} 0 & \text{if } i \neq j \\ k & \text{if } i = j \end{cases}$$

$$(c) X_1, \dots, X_n \text{ generate } D^b(\operatorname{mod-}B) \text{ as a triangulated category}$$

then there is a splendid tilting complex X that lifts C and induces a splendid equivalence between A and B such that, for each $0 \leq i \leq r$, $X_i \cong S_i \otimes_A X$ in $D^b(B)$. \square

Since restriction is a splendid stable equivalence it follows that our complexes satisfy the conditions of 3.5 and so we have:

Corollary 3.6. *The Broué/Rickard conjecture holds for A . \square*

For completeness we now prove that given the information in the appendices, that is the complexes X_i and their homology, then the complexes X_i are independent, up to isomorphism in $K^b(B)$, of the choice of differentials.

Suppose we have constructed two trimmed resolutions of $S_i \otimes_A C$, both consistent with the information in the appendices, up to degree $-n+1$. Moreover, suppose these two complexes are isomorphic and for simplicity we take this isomorphism to be identification.

In constructing the degree $-n$ differential suppose we make two choices $\phi, \phi' \in \operatorname{Hom}_B(P_i^{-n}, K_i^{-n+1})$, where $K_i^{-n+1} = \ker(d^{-n+1})$, and hence two choices for the $-n$ differential. We'll show that for our X_i 's we can find an isomorphism between these two new complexes.

To do this consider the diagram

$$\begin{array}{ccccccc}
P_i^{-n} & & & P_i^{-n+1} & \longrightarrow & P_i^{-n+2} & \longrightarrow \dots \\
& \searrow \phi & & \nearrow j & & & \\
& & K_i^{-n+1} & & & & \\
& & \parallel & & & & \\
P_i^{-n} & & & P_i^{-n+1} & \longrightarrow & P_i^{-n+2} & \longrightarrow \dots \\
& \searrow \phi' & & \nearrow j & & & \\
& & K_i^{-n+1} & & & &
\end{array}$$

where $j : K_i^{-n+1} \rightarrow P_i^{-n+1}$ is the natural inclusion.

If $\text{Hom}_B(P_i^{-n}, H^{-n+1}(X_i)) = 0$, then $\text{im}(\phi)$ is the largest submodule of K_i^{-n+1} for which there are no maps from P_i^{-n} to the quotient. Therefore $\text{im}(\phi) = \text{im}(\phi')$ and we must have an isomorphism ψ such that

$$\begin{array}{ccccccc}
P_i^{-n} & \xrightarrow{d^{-n}=j\phi} & P_i^{-n+1} & \longrightarrow & P_i^{-n+2} & \longrightarrow \dots \\
\downarrow \psi & & \parallel & & \parallel & \\
P_i^{-n} & \xrightarrow{d'^{-n}=j\phi'} & P_i^{-n+1} & \longrightarrow & P_i^{-n+2} & \longrightarrow \dots
\end{array}$$

is a cochain map isomorphism.

It is clear from the appendices that the only complex for which the above does not hold is X_6 ($n = 3$).

Now, fix a map $\phi \in \text{Hom}_B(P_i^{-n}, K_i^{-n+1})$, with $q : K^{-n+1} \rightarrow K^{-n+1}/\text{im}(\phi) = H^{-n+1}(X_i)$ the natural quotient. Suppose there is a map $\eta : P^{-n} \rightarrow K^{-n+1}$ such that $q\eta \neq 0$ and take $\phi' = \lambda\phi + \eta$. If the map η factors as $\eta = \gamma d^{-n} = \gamma j\phi$, where $\gamma : P^{-n+1} \rightarrow K^{-n+1}$, then we can adjust our degree $-n+1$ identification to an isomorphism, $\lambda + j\gamma$, and so extend the cochain isomorphism to

$$\begin{array}{ccccccc}
P_i^{-n} & \xrightarrow{d^{-n}=j\phi} & P_i^{-n+1} & \longrightarrow & P_i^{-n+2} & \longrightarrow \dots \\
\parallel & & \downarrow \lambda+j\gamma & & \parallel & \\
P_i^{-n} & \xrightarrow{d'^{-n}=j\phi'} & P_i^{-n+1} & \longrightarrow & P_i^{-n+2} & \longrightarrow \dots
\end{array}$$

This is the situation that the complex X_6 ($n = 3$) is in and so this shows that as claimed our complexes, X_i , are independent, up to isomorphism in $K^b(B)$, of the choice of differentials.

We note here that we can actually explicitly find the summands of a tilting complex for B giving the derived equivalence above, moreover, we can use this to find the loewy structures of the projective A -modules. We refer the reader to [2] for further details and other examples.

4 Lifting Derived Equivalences

In this section we give a proof of 1.4. We begin with a definition.

Definition 4.1. *Let \tilde{G} be a group with a normal subgroup G and let \tilde{A} be a summand of $k\tilde{G}$ and A be a block of G . We say \tilde{A} and A are linked if every simple \tilde{A} -module is a summand of a simple A -module induced up to \tilde{G} and every simple A -module induced up to \tilde{G} is a semisimple \tilde{A} -module.*

We now show that under certain assumptions a block of kG is always linked to a summand of $k\tilde{G}$. We begin by proving a lemma which we shall need in the proof of proposition 4.3.

Lemma 4.2. *Let \tilde{G} be a group with a normal subgroup G , k be an algebraically closed field of characteristic p , $H = \tilde{G}/G$ be a p' -group and M , N and S be $k\tilde{G}$ -modules, then the short exact sequence*

$$0 \rightarrow M \xrightarrow{\alpha} S \xrightarrow{\beta} N \rightarrow 0$$

is split if and only if the short exact sequence

$$0 \rightarrow M \downarrow_G \xrightarrow{\alpha_G} S \downarrow_G \xrightarrow{\beta_G} N \downarrow_G \rightarrow 0$$

is split, where α_G and β_G are the retrictions of α and β to kG -module homomorphisms.

Proof. The result follows from the fact that since the index of G in \tilde{G} is prime to p , the restriction map:

$$Ext_{k\tilde{G}}^1(N, M) \rightarrow Ext_{kG}^1(N \downarrow_G, M \downarrow_G)$$

is injective. □

Proposition 4.3. *Let \tilde{G} be a group with a normal subgroup G , k be an algebraically closed field of characteristic p and $H = \tilde{G}/G$ be a p' -group, then every simple kG -module induced up to \tilde{G} is semisimple.*

Proof. Let S be a simple kG -module, then by Mackeys decomposition formula $S \uparrow^{\tilde{G}} \downarrow_G$ is semisimple. Suppose $S \uparrow^{\tilde{G}}$ is not semisimple, then there is a non-split short exact sequence $0 \rightarrow M \rightarrow S \uparrow^{\tilde{G}} \rightarrow N \rightarrow 0$, but since $S \uparrow^{\tilde{G}} \downarrow_G$ is semisimple this short exact sequence becomes split when you apply the restriction functor and this contradicts lemma 4.2. □

Proposition 4.4. *Let \tilde{G} be a group with a normal subgroup G , k be an algebraically closed field of characteristic p , $H = \tilde{G}/G$ be a p' -group and A be a block of kG which is stable under the action of H . Then there is a summand \tilde{A} of $k\tilde{G}$ which is linked to A .*

Proof. Let $\tilde{A} = A \uparrow^{\tilde{G}}$, then as A is stable under the action of H it follows that \tilde{A} is a summand of $k\tilde{G}$ as a $(k\tilde{G}, k\tilde{G})$ -bimodule. By 4.3 we know that every simple A -module induced up to \tilde{G} is a semisimple \tilde{A} -module and clearly, every simple \tilde{A} -module is a summand of a simple A -module induced up to \tilde{G} . Therefore \tilde{A} and A are linked. \square

For the rest of this section we take k to be an algebraically closed field of characteristic p , \tilde{G} to be a group with a normal subgroup G , \tilde{N} to be a group with a normal subgroup N and we assume $H = \tilde{G}/G \cong \tilde{N}/N$ is a p' -group. We also take A to a block of kG stable under the action of H , B to be a block of kN also stable under the action of H and set $\tilde{A} = A \uparrow^{\tilde{G}}$ and $\tilde{B} = B \uparrow^{\tilde{N}}$. By 4.4 we know that \tilde{A} and A are linked and \tilde{B} and B are linked. Moreover, we list the simple A -modules as $s_1, \dots, s_m, s_{11}, \dots, s_{1k_1}, s_{21}, \dots, s_{2k_2}, \dots, s_{r1}, \dots, s_{rk_r}$ such that H fixes s_1, \dots, s_m and for each $1 \leq i \leq r$ permutes s_{i1}, \dots, s_{ik_i} , and we denote the simple \tilde{A} -modules as $S_{11}, \dots, S_{1k_1}, \dots, S_{m1}, \dots, S_{mk_m}, \tilde{S}_{11}, \dots, \tilde{S}_{1l_1}, \dots, \tilde{S}_{r1}, \dots, \tilde{S}_{rl_r}$, such that the S_{ij} 's are summands of $s_i \uparrow^{\tilde{G}}$ and the \tilde{S}_{ij} 's are summands of $s_{i1} \uparrow^{\tilde{G}}$. We should note here that $s_{ij} \uparrow^{\tilde{G}} \cong s_{i1} \uparrow^{\tilde{G}}$ for $1 \leq i \leq r$ and $1 \leq j \leq k_i$.

Lemma 4.5. *If $X_1, \dots, X_n \in D^b(\text{mod-}B)$ generate $D^b(\text{mod-}B)$, then $X_1 \uparrow^{\tilde{N}}, \dots, X_n \uparrow^{\tilde{N}} \in D^b(\text{mod-}\tilde{B})$ generate $D^b(\text{mod-}\tilde{B})$ as a thick subcategory.*

Proof. Induction is an exact functor and so it sends triangles to triangles, therefore, since X_1, \dots, X_n generate $D^b(\text{mod-}B)$ it follows that $s \uparrow^{\tilde{N}}$ is in the triangulated category generated by $X_1 \uparrow^{\tilde{N}}, \dots, X_n \uparrow^{\tilde{N}}$, where s is a simple B -module. As \tilde{B} and B are linked we know that each simple \tilde{B} -module is a summand of a module in the triangulated category generated by $X_1 \uparrow^{\tilde{N}}, \dots, X_n \uparrow^{\tilde{N}}$ and so $X_1 \uparrow^{\tilde{N}}, \dots, X_n \uparrow^{\tilde{N}}$ generate $D^b(\text{mod-}\tilde{B})$ as a thick subcategory. \square

Lemma 4.6. *Let $X_1, \dots, X_n \in D^b(\text{mod-}B)$ be a cohomologically schurian set of generators, stable under the action of H such that*

$$\text{Hom}_{D^b(B)/K^b(P_B)}(X_i, X_i) = k$$

and if $i \neq j$ then

$$\text{Hom}_{D^b(B)/K^b(P_B)}(X_i, X_j) = 0.$$

Then, for each $1 \leq i, j \leq n$, the natural map

$$\text{Hom}_{D^b(\tilde{B})}(X_i \uparrow^{\tilde{N}}, X_j \uparrow^{\tilde{N}}) \rightarrow \text{Hom}_{D^b(\tilde{B})/K^b(P_{\tilde{B}})}(X_i \uparrow^{\tilde{N}}, X_j \uparrow^{\tilde{N}})$$

is an isomorphism of rings.

Proof. Clearly the natural map

$$\text{Hom}_{D^b(B)}(X_i, X_i) \rightarrow \text{Hom}_{D^b(B)/K^b(P_B)}(X_i, X_i)$$

is non-zero, since the identity map is mapped to the identity map, and so this map must be an isomorphism of rings since $\text{Hom}_{D^b(B)}(X_i, X_i) = k$ and $\text{Hom}_{D^b(B)/K^b(P_B)}(X_i, X_i) = k$. Also, if $i \neq j$ then

$$\text{Hom}_{D^b(B)}(X_i, X_j) \rightarrow \text{Hom}_{D^b(B)/K^b(P_B)}(X_i, X_j)$$

is trivially an isomorphism of rings since both sides are zero.

Since induction and restriction are adjoints we know that the natural map

$$\text{Hom}_{D^b(\tilde{B})}(X_i \uparrow^{\tilde{N}}, X_j \uparrow^{\tilde{N}}) \rightarrow \text{Hom}_{D^b(\tilde{B})/K^b(P_{\tilde{B}})}(X_i \uparrow^{\tilde{N}}, X_j \uparrow^{\tilde{N}})$$

is an isomorphism of rings if and only if the natural map

$$\text{Hom}_{D^b(B)}(X_i, X_j \uparrow^{\tilde{N}} \downarrow_N) \rightarrow \text{Hom}_{D^b(B)/K^b(P_B)}(X_i, X_j \uparrow^{\tilde{N}} \downarrow_N)$$

is an isomorphism of rings. By Mackey's theorem this is true if and only if the natural map

$$\text{Hom}_{D^b(B)}(X_i, X_j^{h_1} \oplus \cdots \oplus X_j^{h_{|H|}}) \rightarrow \text{Hom}_{D^b(B)/K^b(P_B)}(X_i, X_j^{h_1} \oplus \cdots \oplus X_j^{h_{|H|}})$$

is an isomorphism of rings, where $H = \{h_1, \dots, h_{|H|}\}$ and we are denoting the image of X_i under the action of h by X_i^h , which is true if and only if the natural map

$$\bigoplus_{k=1}^{|H|} \text{Hom}_{D^b(B)}(X_i, X_j^{h_k}) \rightarrow \bigoplus_{k=1}^{|H|} \text{Hom}_{D^b(B)/K^b(P_B)}(X_i, X_j^{h_k})$$

is an isomorphism of rings. This is clearly true since above we've shown it to be true for each summand. \square

We can now prove our result 1.4 which we state again for the reader's convenience.

Theorem 4.7. *Let $\tilde{F} : \overline{\text{mod}} - \tilde{A} \rightarrow \overline{\text{mod}} - \tilde{B}$ and $F : \overline{\text{mod}} - A \rightarrow \overline{\text{mod}} - B$ be stable equivalences of Morita type such that*

$$F(-) \uparrow^{\tilde{N}} \cong \tilde{F}(- \uparrow^{\tilde{G}}) \quad (2)$$

Suppose $X_1, \dots, X_m, X_{11}, \dots, X_{1k_1}, \dots, X_{r1}, \dots, X_{rk_r} \in D^b(\text{mod} - B)$ is a cohomologically schurian set of generators such that H permutes X_{i1}, \dots, X_{ik_i} for $1 \leq i \leq r$ and fixes X_1, \dots, X_m , $X_{ij} \cong F(s_{ij})$ for $1 \leq i \leq r$ and $1 \leq j \leq k_r$, and $X_i \cong F(s_i)$ for $1 \leq i \leq m$. Then \tilde{A} and \tilde{B} are derived equivalent.

Proof. Let $i, j \in \{1, \dots, m, 11, 21, \dots, r1\}$, then by Mackey's theorem we know that

$$\begin{aligned} \text{Hom}_{D^b(\tilde{B})}(X_i \uparrow^{\tilde{N}}, X_j \uparrow^{\tilde{N}} [t]) &\cong \text{Hom}_{D^b(B)}(X_i \uparrow^{\tilde{N}} \downarrow_N, X_j[t]) \cong \\ &\text{Hom}_{D^b(B)}(X_i^{h_1} \oplus \cdots \oplus X_i^{h_{|H|}}, X_j[t]) \end{aligned}$$

where $H = \{h_1, \dots, h_{|H|}\}$, hence

$$\text{Hom}_{D^b(\tilde{B})}(X_i \uparrow^{\tilde{N}}, X_j \uparrow^{\tilde{N}}[t]) = 0 \text{ if } t < 0 \text{ or } i \neq j.$$

Let $t \in \{1, \dots, m, 11, 21, \dots, r1\}$, then by (4), 4.6 and the fact that \tilde{A} and A are linked we know that

$$\begin{aligned} \text{End}_{D^b(\tilde{B})}(X_t \uparrow^{\tilde{N}}) &\cong \text{End}_{D^b(\tilde{B})/K^b(P_{\tilde{B}})}(X_t \uparrow^{\tilde{N}}) \cong \text{End}_{D^b(\tilde{A})/K^b(P_{\tilde{A}})}(\tilde{F}^{-1}(X_t \uparrow^{\tilde{N}})) \\ &\cong \text{End}_{D^b(\tilde{A})/K^b(P_{\tilde{A}})}(s_t \uparrow^{\tilde{G}}) \cong M_{n_1}(k) \oplus \dots \oplus M_{n_{q_t}}(k). \end{aligned}$$

Hence in $D^b(\tilde{B})$ we have

$$X_t \uparrow^{\tilde{N}} \cong \underbrace{Y_1^t \oplus \dots \oplus Y_1^t}_{n_1\text{-terms}} \oplus \underbrace{Y_2^t \oplus \dots \oplus Y_2^t}_{n_2\text{-terms}} \oplus \dots \oplus \underbrace{Y_{q_t}^t \oplus \dots \oplus Y_{q_t}^t}_{n_{q_t}\text{-terms}}$$

such that

$$\text{Hom}_{D^b(\tilde{B})}(Y_i^{t_1}, Y_j^{t_2}[w]) = \begin{cases} k & \text{if } i = j, t_1 = t_2 \text{ and } w = 0 \\ 0 & \text{otherwise.} \end{cases}$$

Moreover, since $X_{ij} \uparrow^{\tilde{N}} \cong X_{i1} \uparrow^{\tilde{N}}$ for $1 \leq i \leq r$ and $1 \leq j \leq k_i$ it follows from 4.5 that the set $Y_1^1, \dots, Y_{q_1}^1, \dots, Y_1^{r1}, \dots, Y_{q_{r1}}^{r1}$ generates $D^b(\text{mod-}\tilde{B})$ as a thick subcategory and so this set is a cohomologically schurian set of generators.

In the stable module category we have the following isomorphisms for $t \in \{1, \dots, m, 11, 21, \dots, r1\}$:

$$Y_1^t \oplus \dots \oplus Y_1^t \oplus \dots \oplus Y_{q_t}^t \oplus \dots \oplus Y_{q_t}^t \cong F(s_t) \uparrow^{\tilde{N}} \cong \tilde{F}(s_t \uparrow^{\tilde{G}})$$

which is isomorphic to:

$$\begin{cases} \tilde{F}(S_{t1}) \oplus \dots \oplus \tilde{F}(S_{t1}) \oplus \dots \oplus \tilde{F}(S_{tk_t}) \oplus \dots \oplus \tilde{F}(S_{tk_t}) & \text{if } t \in \{1, \dots, m\} \\ \tilde{F}(\tilde{S}_{t1}) \oplus \dots \oplus \tilde{F}(\tilde{S}_{t1}) \oplus \dots \oplus \tilde{F}(\tilde{S}_{tl_t}) \oplus \dots \oplus \tilde{F}(\tilde{S}_{tl_t}) & \text{if } t \in \{11, \dots, r1\} \end{cases}$$

therefore, without loss of generality, we can assume that in the stable module category we have

$$Y_i^t \cong \tilde{F}(S_{ti}) \text{ if } t \in \{1, \dots, m\}$$

$$Y_i^t \cong \tilde{F}(\tilde{S}_{ti}) \text{ if } t \in \{11, \dots, r1\}.$$

We have now shown that the set $Y_1^1, \dots, Y_{q_1}^1, \dots, Y_1^{r1}, \dots, Y_{q_{r1}}^{r1} \in D^b(\text{mod-}\tilde{B})$ satisfies the conditions of 1.2 and so \tilde{A} and \tilde{B} are derived equivalent. \square

We note here we could have done all of the above in the context of stable equivalences of splendid type and obtained the following.

Corollary 4.8. *Let $\tilde{F} : \overline{\text{mod}} - \tilde{A} \rightarrow \overline{\text{mod}} - \tilde{B}$ and $F : \overline{\text{mod}} - A \rightarrow \overline{\text{mod}} - B$ be stable equivalences of splendid type such that*

$$F(-) \uparrow^{\tilde{N}} \cong \tilde{F}(- \uparrow^{\tilde{G}}) \quad (3)$$

Suppose $X_1, \dots, X_m, X_{11}, \dots, X_{1k_1}, \dots, X_{r1}, \dots, X_{rk_r} \in D^b(\text{mod} - B)$ is a cohomologically schurian set of generators such that H permutes X_{i1}, \dots, X_{ik_i} for $1 \leq i \leq r$ and fixes X_1, \dots, X_m , $X_{ij} \cong F(s_{ij})$ for $1 \leq i \leq r$ and $1 \leq j \leq k_r$, and $X_i \cong F(s_i)$ for $1 \leq i \leq m$. Then there is a splendid derived equivalence between \tilde{A} and \tilde{B} .

Proof. The proof is exactly the same as that for 4.7, except we find that the set $Y_1^1, \dots, Y_{q_1}^1, \dots, Y_1^{r1}, \dots, Y_{q_r}^{r1} \in D^b(\text{mod} - \tilde{B})$ satisfies the conditions of 3.5 instead of 1.2. \square

To finish this section we show that the conditions (4) and (5) in the above theorems are not vacuous.

Lemma 4.9. *Suppose restriction is a stable equivalence between \tilde{A} and \tilde{B} and A and B , then in the stable module category*

$$- \downarrow_N^G \uparrow_N^{\tilde{N}} \cong - \uparrow_{\tilde{G}}^{\tilde{G}} \downarrow_{\tilde{N}}^{\tilde{G}}$$

Proof. In the stable module category the inverse functor of restriction is induction and so

$$- \downarrow_N^G \uparrow_N^G \cong \text{id}_G(-)$$

and

$$- \uparrow_{\tilde{N}}^{\tilde{G}} \downarrow_{\tilde{N}}^{\tilde{G}} \cong \text{id}_{\tilde{N}}(-)$$

Therefore we have

$$- \downarrow_N^G \uparrow_N^{\tilde{N}} \cong - \downarrow_N^G \uparrow_N^{\tilde{N}} \uparrow_{\tilde{N}}^{\tilde{G}} \downarrow_{\tilde{N}}^{\tilde{G}} \cong - \downarrow_N^G \uparrow_N^{\tilde{G}} \downarrow_{\tilde{N}}^{\tilde{G}} \cong - \downarrow_N^G \uparrow_N^G \uparrow_{\tilde{G}}^{\tilde{G}} \downarrow_{\tilde{N}}^{\tilde{G}} \cong - \uparrow_{\tilde{G}}^{\tilde{G}} \downarrow_{\tilde{N}}^{\tilde{G}}$$

as required. \square

5 Broué's Conjecture for the Group ${}^2F_4(2)$

In this section we combine the results of §3 and §4 to prove that Broué's conjecture holds for the group ${}^2F_4(2)$.

We have an action of C_2 on ${}^2F_4(2)'$ which permutes the simple modules and in fact we have ${}^2F_4(2)' \rtimes C_2 \cong {}^2F_4(2)$. If we use the notation of §3 then the principal block of ${}^2F_4(2)$ is linked to A , restriction is a stable equivalence between it and its Brauer correspondent and $S_i \downarrow_{N_2 F_4(2)'(P)} \uparrow^{N_2 F_4(2)(P)} \cong S_i \uparrow^{2F_4(2)} \downarrow_{N_2 F_4(2)(P)}$ by 4.9, moreover, C_2 permutes the complexes X_i . It is now clear that the conditions of 4.7 are satisfied and so Broué's conjecture holds for the principal block of ${}^2F_4(2)$.

In fact we can explicitly obtain the complexes satisfying the conditions of 1.2 since

$$\begin{aligned}
X_1 \uparrow^{N_{2F_4(2)}(P)} &= X'_1 \oplus X'_2 \\
X_2 \uparrow^{N_{2F_4(2)}(P)} &= X_3 \uparrow^{N_{2F_4(2)}(P)} = X'_3 \\
X_4 \uparrow^{N_{2F_4(2)}(P)} &= X'_4 \oplus X'_5 \\
X_5 \uparrow^{N_{2F_4(2)}(P)} &= X'_6 \oplus X'_7 \\
X_6 \uparrow^{N_{2F_4(2)}(P)} &= X'_8 \oplus X'_9 \\
X_7 \uparrow^{N_{2F_4(2)}(P)} &= X_8 \uparrow^{N_{2F_4(2)}(P)} = X'_{10} \\
X_9 \uparrow^{N_{2F_4(2)}(P)} &= X'_{11} \oplus X'_{12} \\
X_{10} \uparrow^{N_{2F_4(2)}(P)} &= X'_{13} \oplus X'_{14} \\
X_{11} \uparrow^{N_{2F_4(2)}(P)} &= X_{12} \uparrow^{N_{2F_4(2)}(P)} = X'_{15} \\
X_{13} \uparrow^{N_{2F_4(2)}(P)} &= X_{14} \uparrow^{N_{2F_4(2)}(P)} = X'_{16}
\end{aligned}$$

and explicitly the X'_i 's are the complexes:

$$\begin{aligned}
X'_1 &: \cdots \rightarrow RG_1 \\
X'_2 &: \cdots \rightarrow RG_2 \\
X'_3 &: \cdots \rightarrow RG_3 \\
X'_4 &: \cdots \rightarrow P_{10} \rightarrow P_{13} \rightarrow RG_4 \\
X'_5 &: \cdots \rightarrow P_5 \rightarrow P_{11} \rightarrow RG_5 \\
X'_6 &: \cdots \rightarrow P_9 \rightarrow P_{12} \rightarrow P_{12} \oplus P_8 \rightarrow P_8 \oplus P_{16} \rightarrow P_{16} \oplus P_4 \rightarrow P_4 \oplus P_{13} \rightarrow RG_6 \\
X'_7 &: \cdots \rightarrow P_8 \rightarrow P_{14} \rightarrow P_{14} \oplus P_9 \rightarrow P_9 \oplus P_{16} \rightarrow P_{16} \oplus P_3 \rightarrow P_3 \oplus P_{11} \rightarrow RG_7 \\
X'_8 &: \cdots \rightarrow P_4 \rightarrow P_4 \oplus P_{13} \rightarrow P_{10} \oplus P_9 \rightarrow RG_8 \\
X'_9 &: \cdots \rightarrow P_3 \rightarrow P_3 \oplus P_{11} \rightarrow P_5 \oplus P_8 \rightarrow RG_9 \\
X'_{10} &: \cdots \rightarrow P_9 \oplus P_8 \rightarrow P_{16} \rightarrow P_{16} \rightarrow P_9 \oplus P_8 \rightarrow P_7 \rightarrow RG_{10} \\
X'_{11} &: \cdots \rightarrow P_3 \rightarrow P_{11} \rightarrow P_{11} \oplus P_8 \rightarrow P_8 \oplus P_{15} \rightarrow RG_{11} \\
X'_{12} &: \cdots \rightarrow P_4 \rightarrow P_{13} \rightarrow P_{13} \oplus P_9 \rightarrow P_9 \oplus P_{15} \rightarrow RG_{12} \\
X'_{13} &: \cdots \rightarrow P_8 \rightarrow P_7 \oplus P_8 \rightarrow P_7 \oplus P_{14} \rightarrow P_{14} \oplus P_{12} \rightarrow P_{12} \oplus P_{16} \rightarrow RG_{13} \\
X'_{14} &: \cdots \rightarrow P_9 \rightarrow P_9 \oplus P_7 \rightarrow P_7 \oplus P_{12} \rightarrow P_{12} \oplus P_{14} \rightarrow P_{14} \oplus P_{16} \rightarrow RG_{14} \\
X'_{15} &: \cdots \rightarrow P_{12} \oplus P_{14} \rightarrow P_7 \oplus P_8 \oplus P_9 \oplus P_{16} \rightarrow P_{16} \oplus P_{16} \rightarrow \\
&\quad \rightarrow P_{12} \oplus P_{14} \oplus P_3 \oplus P_4 \rightarrow P_7 \oplus P_{11} \oplus P_{13} \rightarrow P_{10} \oplus P_{16} \oplus P_5 \rightarrow RG_{15} \\
X'_{16} &: \cdots \rightarrow P_9 \oplus P_8 \rightarrow P_{16} \rightarrow P_{16} \rightarrow P_9 \oplus P_8 \rightarrow P_{15} \oplus P_7 \rightarrow P_{12} \oplus P_{14} \rightarrow RG_{16}
\end{aligned}$$

Using the Loewy layers of the homology of these complexes, contained in appendix B, and using Holloway's program [9] it is easy to see that these complexes do indeed satisfy the conditions of Rickard's theorem.

Theorem 5.1. *Broué's conjecture holds for ${}^2F_4(2)$.*

Proof. The order of ${}^2F_4(2)$ is $35942400 = 2^{12}3^35^213$, and so we only have to consider $k^2F_4(2)$ for a field k of characteristic 2,3,5 or 13. It turns out, and this can easily be checked using GAP [3], that the only block of $k^2F_4(2)$ in any characteristic with a non-cyclic abelian defect group is the principal block of $k^2F_4(2)$ in characteristic 5, hence this is the only case we have to prove and this is exactly what we have done above. \square

A Green Correspondents

Here we denote the Green correspondent of S_i by RG_i and the projective cover of S_i by P_i .

${}^2F_4(2)'$

The summands of $S_i \downarrow_N$, for the simple ${}^2F_4(2)'$ -modules are listed below, from this the reader could recreate the ordering of the simple N -modules used in this paper.

$$\begin{aligned}
S_1 \downarrow_N &= RG_1 \\
S_2 \downarrow_N &= RG_2 \oplus P_6 \\
S_3 \downarrow_N &= RG_3 \oplus P_4 \\
S_4 \downarrow_N &= RG_4 \\
S_5 \downarrow_N &= RG_5 \\
S_6 \downarrow_N &= RG_6 \oplus P_3 \oplus P_5 \\
S_7 \downarrow_N &= RG_7 \oplus P_{13} \oplus P_1 \\
S_8 \downarrow_N &= RG_8 \oplus P_{13} \oplus P_1 \\
S_9 \downarrow_N &= RG_9 \oplus P_{14} \oplus P_7 \oplus P_{10} \oplus P_{11} \oplus P_{12} \\
S_{10} \downarrow_N &= RG_{10} \oplus P_{14} \oplus P_8 \oplus P_7 \oplus P_9 \oplus P_{11} \\
S_{11} \downarrow_N &= RG_{11} \oplus P_{14} \oplus P_{14} \oplus P_{13} \oplus P_9 \oplus P_{12} \oplus P_2 \oplus P_4 \oplus P_6 \\
S_{12} \downarrow_N &= RG_{12} \oplus P_{14} \oplus P_{14} \oplus P_{13} \oplus P_8 \oplus P_{10} \oplus P_2 \oplus P_4 \oplus P_6 \\
S_{13} \downarrow_N &= RG_{13} \oplus P_{14} \oplus P_{13} \oplus P_{13} \oplus P_7 \oplus P_8 \oplus P_9 \oplus P_{10} \oplus P_{11} \oplus P_{12} \oplus P_3 \oplus P_5 \\
S_{14} \downarrow_N &= RG_{14} \oplus P_{14} \oplus P_{13} \oplus P_{13} \oplus P_7 \oplus P_8 \oplus P_9 \oplus P_{10} \oplus P_{11} \oplus P_{12} \oplus P_3 \oplus P_5
\end{aligned}$$

The Loewy structures of the Green correspondents RG_i are shown below.

$$\begin{array}{ccccccc}
& & & & & 14 & \\
& & & & & 12 & 10 \\
& & & & & 4 & 6 & 13 \\
RG_1 = & 1, & RG_2 = & 4, & RG_3 = & 6, & RG_4 = & 8 & 7 & 9 \\
& & & & & & & 14 & 14 \\
& & & & & & & 11 \\
& & & & & & & 2
\end{array}$$

$$\begin{array}{cccc}
& 2\ 14 & 7\ 11 & \\
& 11\ 7 & 1\ 13 & 3 \\
& 14 & 9\ 8 & 10 & 5 \\
RG_5 = & 10\ 12 & RG_6 = & 3\ 5\ 14 & RG_7 = & 13 & RG_8 = & 12 \\
& 6\ 4\ 13 & & 12\ 11\ 10 & & 8 & & 9 \\
& 9\ 8 & & 13 & & 5 & & 3 \\
& 14 & & 7 & & & & \\
\\
& 7\ 8\ 9 & & 10\ 12\ 13 & & & & \\
& 3\ 5\ 1\ 14\ 14 & & 8\ 7\ 9 & & & & \\
RG_9 = & 11\ 11\ 12\ 10\ 12\ 10 & & 3\ 1\ 5\ 14\ 14 & & & & \\
& 6\ 4\ 2\ 13\ 13\ 13\ 13 & RG_{10} = & 11\ 12\ 12\ 10\ 11\ 10 & & & & \\
& 9\ 7\ 9\ 8\ 7\ 8\ 8\ 9 & & 4\ 2\ 6\ 13\ 13\ 13\ 13 & & & & \\
& 5\ 3\ 1\ 14\ 14 & & 9\ 7\ 8\ 9\ 8\ 7 & & & & \\
& 11\ 10\ 12 & & 1\ 5\ 3\ 14\ 14 & & & & \\
& 13 & & 11\ 12\ 10 & & & & \\
\\
& 11\ 10 & & 11\ 12 & & & & \\
& 2\ 4\ 13\ 13 & & 6\ 2\ 13\ 13 & & & & \\
& 7\ 9\ 9\ 8\ 8\ 7 & & 8\ 7\ 7\ 9\ 8\ 9 & & & & \\
RG_{11} = & 3\ 5\ 1\ 14\ 14\ 14 & RG_{12} = & 3\ 1\ 5\ 14\ 14\ 14 & & & & \\
& 11\ 12\ 10\ 12\ 10\ 11 & & 10\ 11\ 11\ 12\ 12\ 10 & & & & \\
& 2\ 6\ 13\ 13 & & 4\ 2\ 13\ 13 & & & & \\
& 7\ 8 & & 7\ 9 & & & & \\
\\
& 13 & & 13 & & & & \\
& 7\ 8 & & 9\ 7 & & & & \\
RG_{13} = & 1\ 14 & RG_{14} = & 1\ 14 & & & & \\
& 10\ 11 & & 12\ 11 & & & & \\
& 13 & & 13 & & & &
\end{array}$$

${}^2\mathbf{F}_4(2)$

The summands of $S_i \downarrow_N$, for the simple ${}^2F_4(2)$ -modules are listed below, from this the reader could recreate the ordering of the simple N -modules used in this paper.

$$\begin{aligned}
S_1 \downarrow_N &= RG_1 \\
S_2 \downarrow_N &= RG_2 \\
S_3 \downarrow_N &= RG_3 \oplus P_6 \\
S_4 \downarrow_N &= RG_4 \\
S_5 \downarrow_N &= RG_5 \\
S_6 \downarrow_N &= RG_6 \\
S_7 \downarrow_N &= RG_7 \\
S_8 \downarrow_N &= RG_8 \oplus P_7 \\
S_9 \downarrow_N &= RG_9 \oplus P_7 \\
S_{10} \downarrow_N &= RG_{10} \oplus P_{14} \oplus P_{12} \oplus P_2 \oplus P_1
\end{aligned}$$

$$\begin{aligned}
S_{11} \downarrow_N &= RG_{11} \oplus P_{16} \oplus P_{11} \oplus P_{10} \oplus P_9 \\
S_{12} \downarrow_N &= RG_{12} \oplus P_{16} \oplus P_{13} \oplus P_8 \oplus P_5 \\
S_{13} \downarrow_N &= RG_{13} \oplus P_{15} \oplus P_{11} \oplus P_{10} \oplus P_9 \\
S_{14} \downarrow_N &= RG_{14} \oplus P_{15} \oplus P_{13} \oplus P_8 \oplus P_5 \\
S_{15} \downarrow_N &= RG_{15} \oplus P_{16} \oplus P_{15} \oplus P_{14} \oplus 2.P_{13} \oplus P_{12} \oplus 2.P_{11} \oplus 2.P_6 \oplus P_4 \oplus P_3 \\
S_{16} \downarrow_N &= RG_{16} \oplus 2.P_{16} \oplus 2.P_{15} \oplus 2.P_{14} \oplus P_{13} \oplus 2.P_{12} \oplus P_{11} \oplus P_{10} \oplus P_9 \oplus P_8 \oplus 2.P_7 \oplus P_5
\end{aligned}$$

The Loewy structures of the Green correspondents RG_i are shown below.

$$\begin{array}{ccccccc}
& & & & 13 & & 11 \\
& & & & 16 & & 16 \\
& & & & 6 \ 12 & & 6 \ 14 \\
RG_1 = 1 & RG_2 = 2 & RG_3 = 6 & RG_4 = & 9 \ 15 & RG_5 = & 8 \ 15 \\
& & & & 13 \ 11 & & 13 \ 11 \\
& & & & 5 & & 10 \\
& & & & 3 & & 4 \\
\\
& & 4 \ 13 & & 3 \ 11 & & 9 \ 10 & & 5 \ 8 \\
& & 8 \ 10 & & 9 \ 5 & & 1 \ 14 & & 2 \ 12 \\
& & 11 & & 13 & & 15 & & 15 \\
RG_6 = & 16 & RG_7 = & 16 & RG_8 = & 13 \ 7 & RG_9 = & 11 \ 7 \\
& 6 \ 14 & & 6 \ 12 & & 10 \ 16 & & 5 \ 16 \\
& 15 & & 15 & & 12 & & 14 \\
& 13 & & 11 & & 9 & & 8 \\
\\
& & & 8 \ 15 & & & 9 \ 15 & & \\
& & 7 & & 2 \ 7 \ 13 \ 11 & & 1 \ 7 \ 13 \ 11 & & \\
& & 16 & & 5 \ 10 \ 16 \ 16 & & 5 \ 10 \ 16 \ 16 & & \\
RG_{10} = & 12 \ 14 & RG_{11} = & 4 \ 6 \ 12 \ 14 \ 12 \ 14 & RG_{12} = & 3 \ 6 \ 12 \ 14 \ 14 \ 12 & & \\
& 15 & & 9 \ 8 \ 15 \ 15 \ 15 & & 9 \ 8 \ 15 \ 15 \ 15 & & \\
& 7 & & 2 \ 7 \ 11 \ 13 & & 2 \ 7 \ 11 \ 13 & & \\
& & & 5 \ 16 & & 5 \ 16 & & \\
& & & 12 & & 12 & & \\
\\
& & 12 \ 16 & & & 14 \ 16 & & \\
& & 9 \ 15 & & & 8 \ 15 & & \\
& & 1 \ 7 \ 13 \ 11 & & & 2 \ 7 \ 13 \ 11 & & \\
RG_{13} = & 5 \ 10 \ 16 \ 16 & RG_{14} = & 5 \ 10 \ 16 \ 16 & & & & \\
& 3 \ 6 \ 14 \ 12 \ 14 \ 12 & & 3 \ 6 \ 14 \ 12 \ 14 \ 12 & & & & \\
& 9 \ 8 \ 15 \ 15 & & 9 \ 8 \ 15 \ 15 & & & & \\
& 2 \ 7 \ 13 \ 11 & & 1 \ 7 \ 13 \ 11 & & & & \\
& 5 \ 16 & & 10 \ 16 & & & &
\end{array}$$

$$\begin{array}{ccccc}
& & 5 & 10 & 16 \\
& & 4 & 3 & 6 & 14 & 12 & 14 & 12 \\
& & 8 & 9 & 8 & 9 & 15 & 15 & 15 & 15 \\
RG_{15} & 1 & 2 & 7 & 7 & 13 & 11 & 11 & 13 & 13 & 11 & RG_{16} & 2 & 1 & 13 & 11 \\
& & 10 & 10 & 5 & 5 & 16 & 16 & 16 & 16 \\
& & 4 & 3 & 6 & 12 & 14 & 12 & 14 \\
& & 9 & 8 & 15
\end{array}$$

B Cohomology Groups of the X'_i s

${}^2\mathbf{F}_4(2)'$

Here we display the Loewy structures of the homology of X_i for $1 \leq i \leq 14$. The reader should refer to this when reading lemma 3.2. Also, if one wanted to recreate the complexes X_i then this information would be sufficient to produce the maps I have used.

$$\begin{array}{l}
H^0(X_1) = 1, \quad H^0(X_2) = 4, \quad H^0(X_3) = 6, \\
H^{-2}(X_4) = 11, \quad H^{-1}(X_4) = 1, \\
H^{-6}(X_5) = 7, \quad H^{-5}(X_5) = 12, \quad H^{-4}(X_5) = \begin{array}{c} 6 \ 1 \ 4 \\ 9 \ 8 \end{array}, \quad H^{-3}(X_5) = \begin{array}{c} 4 \ 6 \\ 9 \ 8 \end{array}, \\
H^{-3}(X_6) = \begin{array}{c} 11 \\ 2 \end{array}, \quad H^{-2}(X_6) = \begin{array}{c} 14 \\ 11 \\ 2 \end{array}, \\
H^{-5}(X_7) = \begin{array}{c} 6 \ 9 \\ 3 \ 8 \\ 14 \end{array}, \quad H^{-4}(X_7) = \begin{array}{c} 11 \\ 2 \end{array}, \quad H^{-3}(X_7) = \begin{array}{c} 1 \\ 11 \end{array}, \quad H^{-2}(X_7) = \begin{array}{c} 6 \\ 8 \end{array}, \\
H^{-5}(X_8) = \begin{array}{c} 4 \ 8 \\ 5 \ 9 \\ 14 \end{array}, \quad H^{-4}(X_8) = \begin{array}{c} 11 \\ 2 \end{array}, \quad H^{-3}(X_8) = \begin{array}{c} 1 \\ 11 \end{array}, \quad H^{-2}(X_8) = \begin{array}{c} 4 \\ 9 \end{array}, \\
H^{-4}(X_9) = \begin{array}{c} 11 \\ 2 \end{array}, \quad H^{-3}(X_9) = \begin{array}{c} 1 \\ 11 \end{array}, \quad H^{-2}(X_9) = 1,
\end{array}$$

$$H^{-5}(X_{10}) = \begin{array}{c} 4 \ 6 \\ 9 \ 8 \\ 14 \ 14, \\ 10 \ 11 \ 12 \\ 2 \ 13 \\ 7 \end{array} \quad H^{-4}(X_{10}) = \begin{array}{c} 14, \\ 11 \\ 2 \end{array}$$

$$H^{-3}(X_{10}) = \begin{array}{c} 14 \\ 11, \\ 2 \end{array} \quad H^{-2}(X_{10}) = 4 \ 6, \quad H^{-1}(X_{10}) = 4 \ 6,$$

$$H^{-6}(X_{11}) = \begin{array}{c} 1 \ 4 \\ 9 \\ 14, \\ 11 \ 12 \\ 13 \end{array} \quad H^{-5}(X_{11}) = \begin{array}{c} 4 \ 1 \\ 9 \ 11 \\ 14, \\ 11 \\ 2 \end{array} \quad H^{-4}(X_{11}) = \begin{array}{c} 8 \\ 14 \\ 11, \\ 2 \end{array}$$

$$H^{-6}(X_{12}) = \begin{array}{c} 1 \ 6 \\ 8 \\ 14, \\ 11 \ 10 \\ 13 \end{array} \quad H^{-5}(X_{12}) = \begin{array}{c} 6 \ 1 \\ 8 \ 11 \\ 14, \\ 11 \\ 2 \end{array} \quad H^{-4}(X_{12}) = \begin{array}{c} 9 \\ 14 \\ 11, \\ 2 \end{array}$$

$$H^{-6}(X_{13}) = \begin{array}{c} 4 \ 8 \\ 5 \ 9 \\ 14 \\ 12 \ 11, \\ 2 \ 13 \\ 7 \end{array} \quad H^{-5}(X_{13}) = \begin{array}{c} 11 \\ 2, \end{array} \quad H^{-4}(X_{13}) = \begin{array}{c} 1 \\ 11, \end{array} \quad H^{-1}(X_{13}) = 4,$$

$$H^{-6}(X_{14}) = \begin{array}{c} 6 \ 9 \\ 3 \ 8 \\ 14 \\ 10 \ 11, \\ 2 \ 13 \\ 7 \end{array} \quad H^{-5}(X_{14}) = \begin{array}{c} 11 \\ 2, \end{array} \quad H^{-4}(X_{14}) = \begin{array}{c} 1 \\ 11, \end{array} \quad H^{-1}(X_{14}) = 6$$

We also have the following short exact sequences, the reader should also refer to this when reading lemma 3.2:

$$0 \rightarrow 11 \rightarrow H^{-2}(X_4) \rightarrow H^0(X_1) \rightarrow 0 \quad (\text{B.1})$$

$$0 \rightarrow 2 \rightarrow H^{-4}(X_9) \rightarrow 11 \rightarrow 0 \quad (\text{B.2})$$

$$0 \rightarrow H^{-4}(X_9) \rightarrow H^{-2}(X_6) \rightarrow 14 \rightarrow 0 \quad (\text{B.3})$$

$$0 \rightarrow 8 \rightarrow H^{-2}(X_7) \rightarrow 6 \rightarrow 0 \quad (\text{B.4})$$

$$0 \rightarrow 9 \rightarrow H^{-2}(X_8) \rightarrow 4 \rightarrow 0 \quad (\text{B.5})$$

$$0 \rightarrow 7 \rightarrow H^{-6}(X_5) \rightarrow 2 \rightarrow 0 \quad (\text{B.6})$$

$$0 \rightarrow H^{-6}(X_5) \rightarrow H^{-5}(X_7) \rightarrow N_1 \rightarrow 0 \quad (\text{B.7})$$

$$0 \rightarrow N_2 \rightarrow H^{-6}(X_{12}) \rightarrow 1 \rightarrow 0 \quad (\text{B.8})$$

$$0 \rightarrow N_2 \rightarrow N_1 \rightarrow N_3 \rightarrow 0 \quad (\text{B.9})$$

$$0 \rightarrow H^{-6}(X_5) \rightarrow H^{-5}(X_8) \rightarrow N_4 \rightarrow 0 \quad (\text{B.10})$$

$$0 \rightarrow N_5 \rightarrow H^{-6}(X_{11}) \rightarrow 1 \rightarrow 0 \quad (\text{B.11})$$

$$0 \rightarrow N_5 \rightarrow N_4 \rightarrow N_6 \rightarrow 0 \quad (\text{B.12})$$

$$0 \rightarrow N_7 \rightarrow H^{-5}(X_7) \rightarrow N_3 \rightarrow 0 \quad (\text{B.13})$$

$$0 \rightarrow N_7 \rightarrow H^{-5}(X_{10}) \rightarrow N_8 \rightarrow 0 \quad (\text{B.14})$$

where the loewy structures of the N_i 's are:

$$\begin{array}{cccc} N_1 = \begin{array}{c} 6 \ 9 \\ 3 \ 8 \\ 14, \\ 10 \ 11 \\ 13 \end{array} & N_2 = \begin{array}{c} 6 \\ 8 \\ 14, \\ 10 \ 11 \\ 13 \end{array} & N_3 = \begin{array}{c} 9, \\ 3 \end{array} & N_4 = \begin{array}{c} 4 \ 8 \\ 5 \ 9 \\ 14, \\ 12 \ 11 \\ 13 \end{array} \\ \\ N_5 = \begin{array}{c} 4 \\ 9 \\ 14, \\ 12 \ 11 \\ 13 \end{array} & N_6 = \begin{array}{c} 8, \\ 5 \end{array} & N_7 = \begin{array}{c} 6 \\ 8 \\ 14, \\ 10 \ 11 \\ 2 \ 13 \\ 7 \end{array} & N_8 = \begin{array}{c} 4 \\ 9, \\ 14 \\ 12 \end{array} \end{array}$$

${}^2\mathbf{F}_4(2)$

Here we display the Loewy structures of the homology of X_i for $1 \leq i \leq 16$. The reader should refer to this when reading §5. Also, if one wanted to recreate the complexes X_i then this information would be sufficient to produce the maps I have used.

$$\begin{array}{ccccccc} H^0(X_1) = & 1 & H^0(X_2) = & 2 & H^0(X_3) = & 6 & \\ H^{-2}(X_4) = & 1 & H^{-1}(X_4) = & 1 & H^{-2}(X_5) = & 2 & H^{-1}(X_5) = & 2 \\ & 10 & & & & 5 & & \\ & & & & & 2 \ 6 & & 6 \\ & & & & & 15 & & 15 \\ H^{-6}(X_6) = & 3 & H^{-5}(X_6) = & 2 \ 3 & H^{-4}(X_6) = & 13 & H^{-3}(X_6) = & 13 \\ & 9 & & & & 10 & & 10 \\ & & & & & 4 & & 4 \end{array}$$

$$H^{-6}(X_7) = \begin{array}{c} 4 \\ 8 \end{array} \quad H^{-5}(X_7) = \begin{array}{c} 1 \ 4 \end{array} \quad H^{-4}(X_7) = \begin{array}{c} 1 \ 6 \\ 15 \\ 11 \\ 5 \\ 3 \end{array} \quad H^{-3}(X_7) = \begin{array}{c} 6 \\ 15 \\ 11 \\ 5 \\ 3 \end{array}$$

$$H^{-3}(X_8) = \begin{array}{c} 10 \\ 4 \end{array} \quad H^{-2}(X_8) = \begin{array}{c} 13 \\ 10 \\ 4 \end{array} \quad H^{-3}(X_9) = \begin{array}{c} 5 \\ 3 \end{array} \quad H^{-2}(X_9) = \begin{array}{c} 11 \\ 5 \\ 3 \end{array}$$

$$H^{-5}(X_{10}) = \begin{array}{c} 6 \ 15 \\ 15 \ 7 \\ 13 \ 11 \\ 5 \ 10 \ 16 \\ 4 \ 3 \ 12 \ 14 \\ 8 \ 9 \end{array}$$

$$H^{-4}(X_{10}) = \begin{array}{c} 5 \ 10 \\ 3 \ 4 \end{array} \quad H^{-3}(X_{10}) = \begin{array}{c} 1 \ 2 \\ 5 \ 10 \end{array} \quad H^{-2}(X_{10}) = \begin{array}{c} 6 \\ 15 \end{array}$$

$$H^{-4}(X_{11}) = \begin{array}{c} 5 \\ 3 \end{array} \quad H^{-3}(X_{11}) = \begin{array}{c} 2 \\ 5 \end{array} \quad H^{-2}(X_{11}) = \begin{array}{c} 2 \end{array}$$

$$H^{-4}(X_{12}) = \begin{array}{c} 10 \\ 4 \end{array} \quad H^{-3}(X_{12}) = \begin{array}{c} 1 \\ 10 \end{array} \quad H^{-2}(X_{12}) = \begin{array}{c} 1 \end{array}$$

$$H^{-5}(X_{13}) = \begin{array}{c} 6 \\ 15 \\ 11 \ 13 \\ 10 \ 16 \\ 4 \ 14 \\ 8 \end{array}$$

$$H^{-4}(X_{13}) = \begin{array}{c} 13 \\ 10 \\ 4 \end{array} \quad H^{-3}(X_{13}) = \begin{array}{c} 13 \\ 10 \\ 4 \end{array} \quad H^{-2}(X_{13}) = \begin{array}{c} 6 \end{array} \quad H^{-1}(X_{13}) = \begin{array}{c} 6 \end{array}$$

$$H^{-5}(X_{14}) = \begin{array}{c} 6 \\ 15 \\ 11 \ 13 \\ 5 \ 16 \\ 3 \ 12 \\ 9 \end{array}$$

$$H^{-4}(X_{14}) = \begin{array}{c} 11 \\ 5 \\ 3 \end{array} \quad H^{-3}(X_{14}) = \begin{array}{c} 11 \\ 5 \\ 3 \end{array} \quad H^{-2}(X_{14}) = 6 \quad H^{-1}(X_{14}) = 6$$

$$H^{-6}(X_{15}) = \begin{array}{c} 2 \ 1 \ 6 \\ 15 \\ 11 \ 13 \\ 10 \ 5 \ 16 \\ 14 \ 12 \end{array} \quad H^{-5}(X_{15}) = \begin{array}{c} 1 \ 2 \ 6 \\ 10 \ 5 \ 15 \\ 11 \ 13 \\ 10 \ 5 \\ 3 \ 4 \end{array} \quad H^{-4}(X_{15}) = \begin{array}{c} 15 \\ 13 \ 11 \\ 10 \ 5 \\ 3 \ 4 \end{array}$$

$$H^{-6}(X_{16}) = \begin{array}{c} 6 \ 15 \\ 15 \ 7 \\ 11 \ 13 \\ 5 \ 10 \ 16 \\ 3 \ 4 \ 12 \ 14 \\ 9 \ 8 \end{array} \quad H^{-5}(X_{16}) = \begin{array}{c} 5 \ 10 \\ 3 \ 4 \end{array} \quad H^{-4}(X_{16}) = \begin{array}{c} 1 \ 2 \\ 5 \ 10 \end{array} \quad H^{-1}(X_{16}) = 6$$

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